MATHEMATICS LEARNING AREA

YEAR 12 MATHEMATICS METHODS UNIT 3

Assessment type: Response

TASK 3 - TEST 2

CALCULATOR-FREE

Syllabus content : identify anti-differentiation as the reverse of differentiation (3.2.1-3.2.9)

Student Name: _

TIME ALLOWED FOR THIS PAPER

Suggested:
Reading and Working time for Cal Free paper: 25 minutes in class under test conditions

MATERIAL REQUIRED / RECOMMENDED FOR THIS PAPER

TO BE PROVIDED BY THE SUPERVISOR
Question/answer booklet

TO BE PROVIDED BY THE CANDIDATE
Standard Items: pens, pencils, pencil sharpener, highlighter, eraser, ruler, drawing templates, Calculator

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be attempted	Suggested working time (minutes)	Marks available
Calculator Free	3	3	25	**40**
			Marks available:	**/40**
			Task Weighting	7% for the pair of units

Instructions to candidates

Question 1 **(13 marks)**

Consider $\frac{dy}{dx} = f(x)$, this is called a differential equation, more specifically a first order differential equation. To solve or find the general solution of a differential equation is to do the reverse of differentiation.
To find the general solution of a differential equation, then some additional information must be required.

Solve the following differential equations to find the general solution or particular solution.

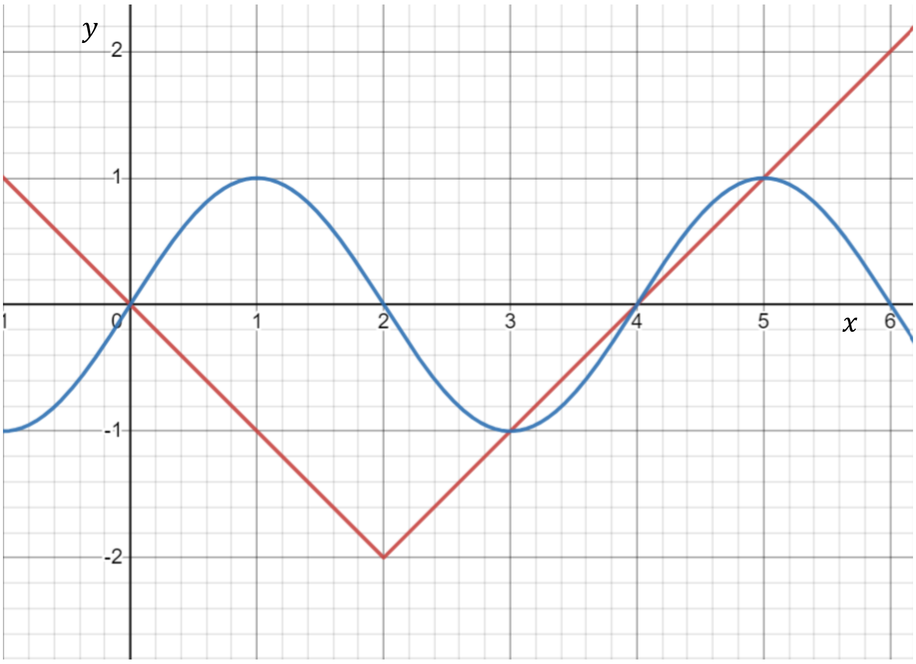
a) $\frac{dy}{dx} = 3x^2 + (x+1)(x^2+2x+1)^2 + e$ **(3 marks)**

b) $\frac{dy}{dx} = \frac{13x}{\sqrt{4x^2-7}}\ at\ (2, \frac{3}{4})$ **(4 marks)**

c) $\frac{dx}{dy} = \frac{3}{(\tan^2 x + 1 + e^{4x} + \frac{3}{\sqrt[3]{3x+4}})}$ $(Hint\colon manipulate\ the\ Pythagorean\ identity)$ **(6 marks)**

Question 2

Determine the exact area of the region trapped by $y = \sin\frac{1}{2}\pi x$ and $y = |x-2| - 2$, from the origin to $x = 3$. **(8 marks)**



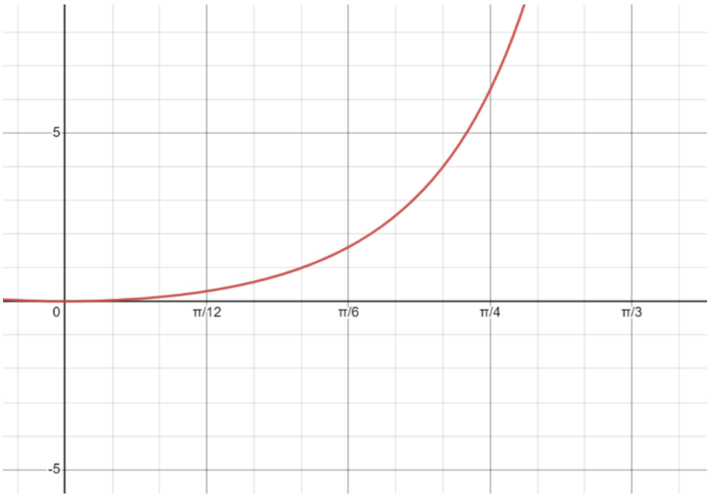
Question 3 **(12 marks)**

The function is given as $y = \frac{x}{\cos^2 x} + x \sin x$.

a) Find $\frac{d}{dx}(\frac{x}{\cos^2 x} + x \sin x)$ **(2 marks)**

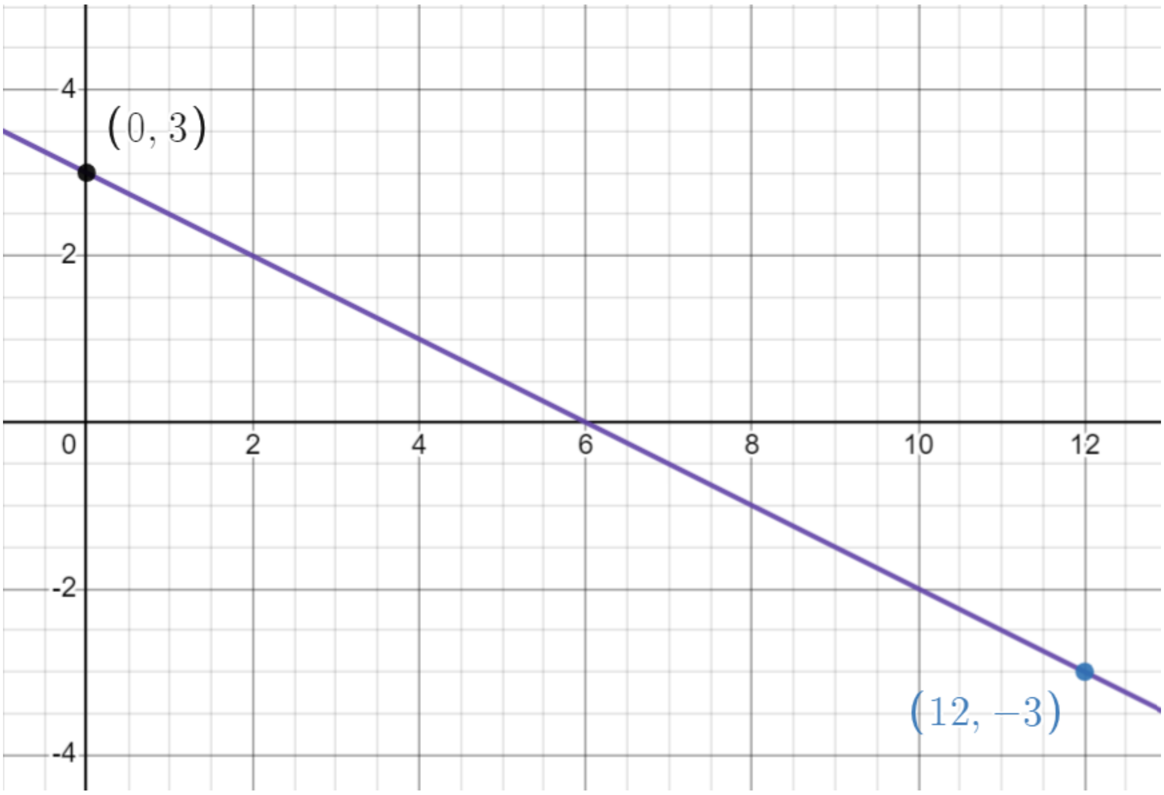
b) Hence, by using $\frac{d}{dx}(x \sin x)$, find $\int \frac{4x \tan x}{\cos^2 x} dx$. **(6 marks)**

c) Hence find the exact area of the function $f(x) = \frac{4x \tan x}{\cos^2 x}$ between the equations $x = 0$ and $x = \frac{\pi}{4}$ **(2 marks)**



Question 4 **(7 marks)**

Shown below is a graph of $y = f(t)$, and consider only by the interval of $[0,12]$



A man saw this graph and said he wanted to make another equation relating to this, but involving with derivatives and integrals. He came up with the equation of $F(x)$, where…

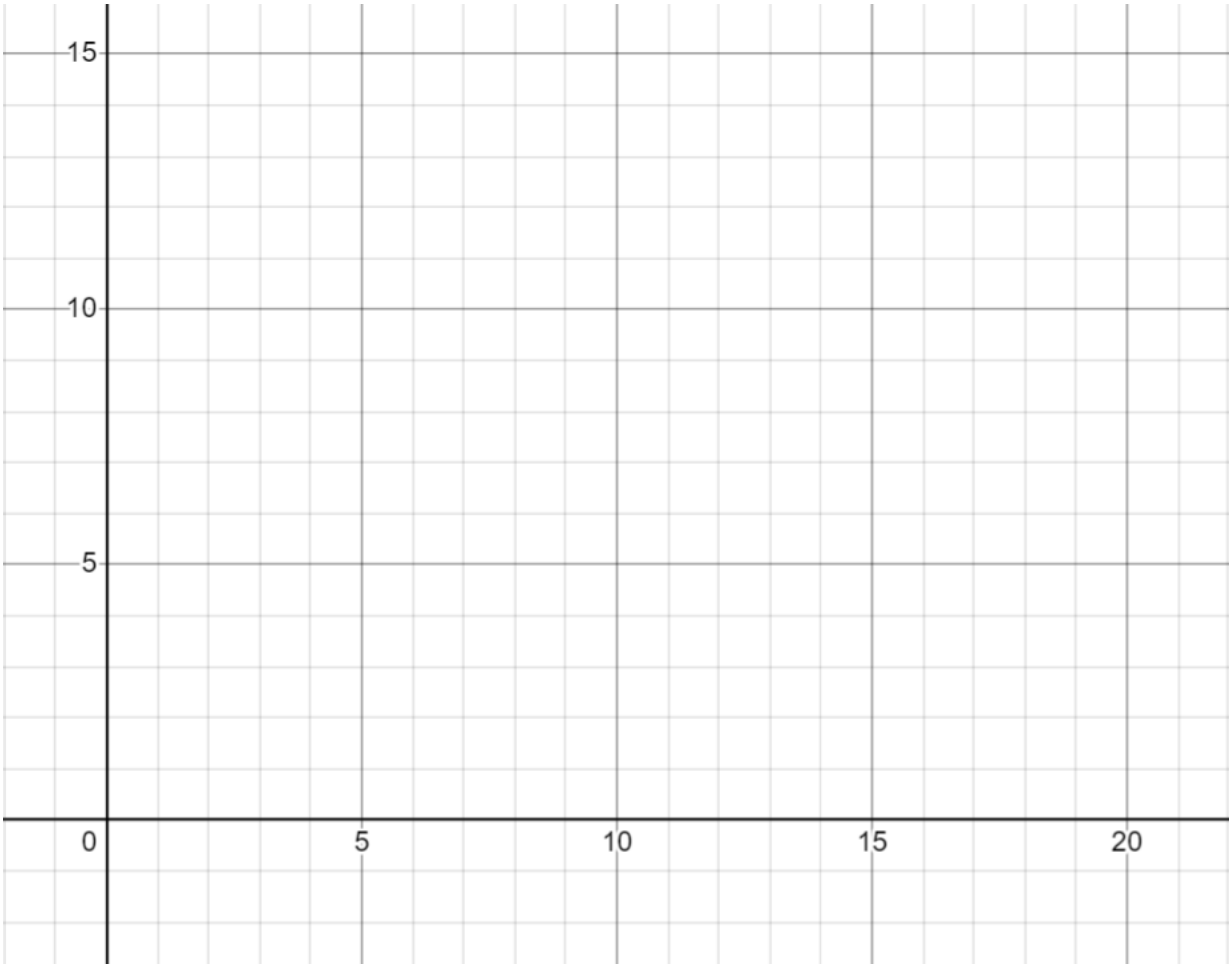
$$F(x) = \frac{d}{dx}\left(\int_0^x f(t)\,dt\right)$$

Explain by using the knowledge of the fundamental theorem of calculus, why this doesn't change anything. **(2 marks)**

The man then wanted to make another equation $A(x)$, where…

$$A(x) = \int_0^x f(t)\,dt$$

Sketch $A(x)$ below, labelling the local maximum/minimum, the coordinates at $x = 6$ and $x = 12$ for the given interval $[0,12]$. **(5 marks)**



END OF CALCULATOR-FREE

Additional working space

Question number: ___________